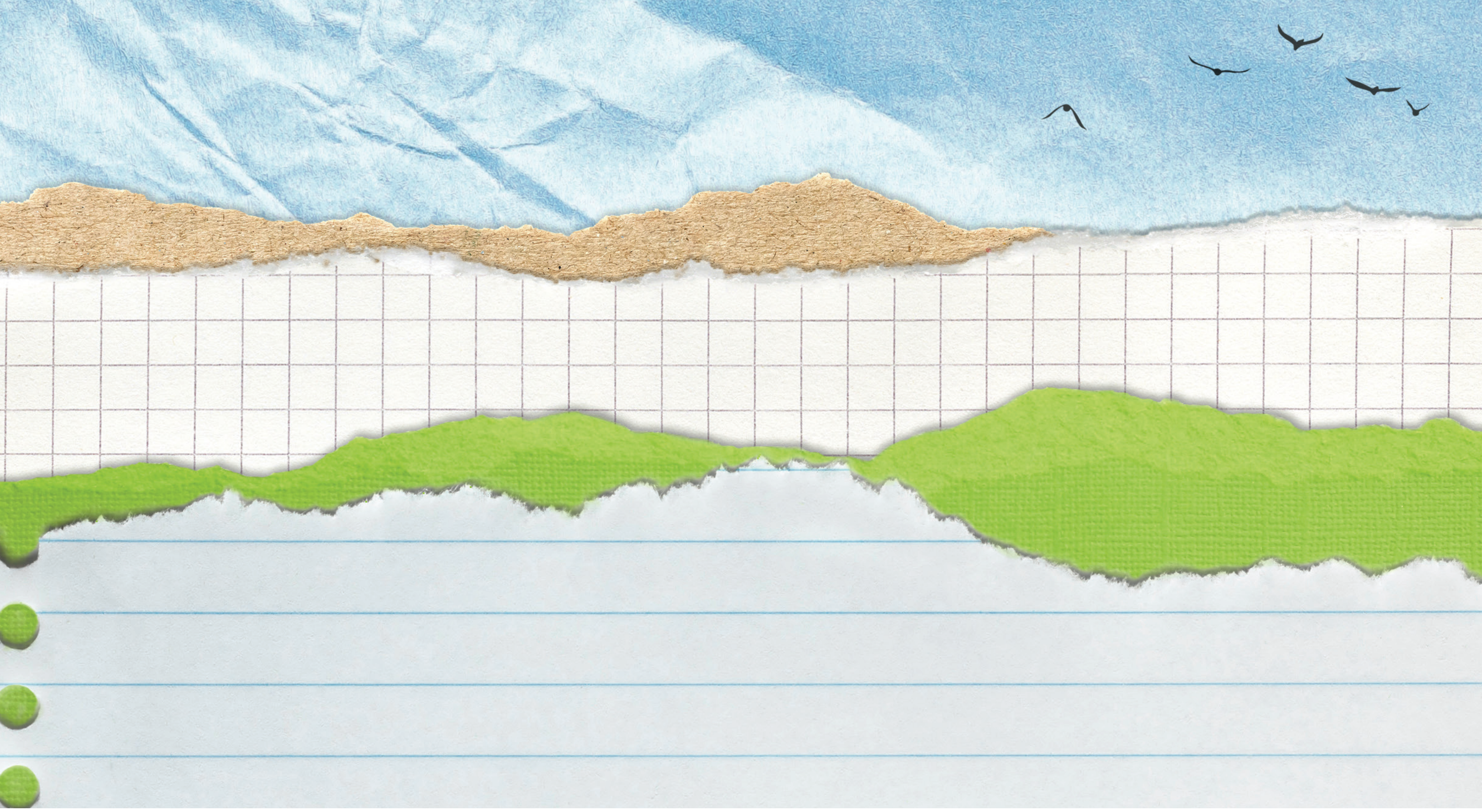


MAXIMIZING THE POTENTIAL OF *Mathematical Writing Prompts*

Learn how to identify, adapt, and create writing prompts to capitalize on the insights you gain about each of your student's thinking.

Tutita M. Casa, Cindy M. Gilson, Micah N. Bruce-Davis,
E. Jean Gubbins, Stacy M. Hayden, and Elizabeth J. Canavan

Whole-class and small-group talk has long allowed educators to gauge students' thinking and understanding of mathematical ideas. Writing further has the potential to provide a unique opportunity to gain additional insights into each individual's perspectives. Although the National Council of Teachers of Mathematics (NCTM), the National Council of Supervisors of Mathematics (NCSM), and the Association of State Supervisors of Mathematics (ASSM) emphasized that "students *did* learn during the pandemic" (2021, p. 1), this event caused us to recognize the importance of carefully considering the writing prompts we provide to our students because we were not always able to drop in on their conversations or facilitate discussions to truly understand students' thinking during remote learning.

Further, students' mathematical talk is essential in the formative assessment process that allows teachers to clarify goals for student learning, elicit evidence of their learning, interpret students' work, and act on these conclusions to guide their subsequent teaching (The Regents of the University of California 2021). Some students may hesitate to participate in oral discourse because, for example, they are afraid of being wrong in front of their peers, so they may not benefit equally from engaging in class discussions (Casa, Choppin, and Moschkovich 2020). More reticent students could have a stronger "voice" when asked to write, which is a personalized and broader way of communicating and participating in the classroom discourse community. Additionally, "writing has the potential to include multiple forms of inscriptions, providing increased avenues for students to represent and construct meaning, which may have implications for equitable opportunities to learn from and participate in mathematics discourse" (Casa, Choppin, and Moschkovich 2020, p. 1). Such an emphasis on writing reflects productive mindsets and practices called for by NCTM, NCSM, and ASSM (2021), including a belief "that all students are mathematically brilliant," and disrupting the approach to teaching that "values only one solution pathway and only some types of knowledge" (p. 5). Ladson-Billings (1997) echoed this belief long before the pandemic when she described a teacher who "helped her students understand that they were knowledgeable and capable of answering questions posed by themselves and others" (p. 703).

We take the same position as Ladson-Billings (1997) and envision the knowledge students are capable of sharing when the utility of writing prompts is maximized. Therefore, this article aims to share five practical approaches for how teachers can identify, adapt, and create their own writing prompts they assign to students. We present the purposes of the approaches, provide examples and nonexamples of each approach across grade levels, share samples of student work, and discuss how student writing can augment teachers' formative assessment process to realize the depth of their students' thinking more comprehensively. Before doing so, we define "mathematical thinking" to reflect the kind of responses we have sought to elicit from students through written prompts and inform decisions teachers can make when applying the five approaches to their contexts.

DEFINING "MATHEMATICAL THINKING"

On the basis of our revisions of an existing unit (Cole et al. 2019), which was one task contributing to a national

Tutita M. Casa, she/her, tutita.casa@uconn.edu, is an associate professor in the Department of Curriculum and Instruction in the Neag School of Education at the University of Connecticut in Storrs and Co-Principal Investigator of Thinking Like Mathematicians: Challenging All Grade 3 Students. She is interested in discourse, including mathematical writing, and supports teachers through curriculum interventions and professional development.

Cindy M. Gilson, she/her, cgilson@uncc.edu, is an associate professor of Gifted Education at the University of North Carolina at Charlotte. Her scholarly work centers on differentiated curriculum and instruction, classroom discourse, teachers' listening orientations, and differentiated professional learning.

Micah N. Bruce-Davis, she/her, micah.bruce-davis@louisiana.edu, teaches K–5 mathematics methods courses and supports residents completing their final year of their teacher preparation program. She seeks ways to support preservice and in-service teachers in implementing classroom practices to develop and support gifted behaviors.

E. Jean Gubbins, ejean.gubbins@uconn.edu, is a professor at the University of Connecticut Department of Educational Psychology. She is the Associate Director for the National Center for Research on Gifted Education and Principal Investigator of Thinking Like Mathematicians: Challenging All Grade 3 Students.

Stacy M. Hayden, she/her, stacy.hayden@uconn.edu, is a doctoral candidate at the University of Connecticut and a research associate for Shaffer Evaluation Group in Williamsburg, Virginia. Her scholarly work centers on equity in gifted education and academic risk taking.

Elizabeth J. Canavan, she/her, elizabeth.canavan@uconn.edu, teaches eighth-grade mathematics at CREC Ana Grace Academy of the Arts Middle School in Bloomfield, Connecticut. She was a graduate research assistant for Thinking Like Mathematicians: Challenging All Grade 3 Students, and her research interests include equity and ethnomathematics.

doi:10.5951/MTLT.2021.0287

curriculum implementation project, we recognize mathematical thinking is challenging to conceptualize and apply. Additionally, due to typical pressures from high-stakes testing and mandated curricula, teachers may rush students to the metaphorical finish line defined by correct solution pathways and answers. Doing so is akin to spoiling the best parts of a movie for a good friend who has yet to watch it.

Instead, allowing all students the opportunities to reason, problem solve, justify, conjecture, and argue *themselves* (NCTM 2000, 2014; NGA Center and CCSSO 2010) is important and more equitable. These national standards subtly highlight another key aspect of students' mathematical thinking, which essentially indicates a verb, not a noun. Gutiérrez (2018) emphasizes how engaging students in these practices and processes is essential for rehumanizing mathematics and strengthening students' mathematical identities so that they have the opportunity to engage with the living practice of mathematics continuously. There should not be only one path that students take, and, instead, we should promote mathematics as "full of . . . power dynamics, debates, divergent answers, and rule breaking" (p. 5). In summary, mathematical thinking is analogous to the emotions we experience when watching movies, whereas the movie's topic is analogous to the mathematical content. Just like our experiences watching a movie are integral to the movie's topic, mathematical thinking is integral to the mathematical content.

IMPLEMENTING FIVE APPROACHES TO IDENTIFY AND ADAPT WRITING PROMPTS

The following approaches to identify, adapt, and create writing prompts are grounded in the notion of "mathematical writing" that calls "for students *to reason* mathematically" (Casa et al. 2016, p. 3, original emphasis) that similarly encapsulates the mathematical thinking teachers can rely on to formatively assess their students.

1. *Promote Students' Solution Paths* addresses the necessity of student agency.
2. *Go beyond Asking Students* to Simply "Explain" calls for deeper conceptual explanations versus simply listing computation steps.
3. *Prompt Students to Share Their Reasoning* offers students the opportunity to communicate their mathematical reasoning to argue the soundness of their own solution and process.

Approaches 1–3 prompt individual students' mathematical solutions, whereas the next two approaches focus on prompting students to apply what they have learned through the evaluation of others' solutions.

4. *Have Students Consider the Validity of a Given Solution* frames the writing prompts to draw out students' ability to evaluate the validity of someone else's solution.
5. *Have Students Debate the Validity of Two Given Solutions* has students weigh in on a debate to prove or disprove others' solutions. Writing prompts may also address multiple approaches.

Applying Renzulli and Waicunas's (2016) view of infusion, teachers can identify and use existing components from their materials to increase opportunities for students to think and write mathematically. These opportunities might be call-out boxes on the side of workbook pages, problem-solving sections at the end of a chapter, or companion challenge materials. The topics addressed in these extension components attend to the same content standards as the main sections in these materials, yet they are more likely to position students as mathematical thinkers and writers.

In addition to applying the following approaches to identify viable writing prompts within existing materials, one might also use them to adapt writing prompts. Given that most writing prompts included in grade 3 student-facing pages across 10 commercially available series ask students to explain what they did rather than explain why (Casa et al. 2019), teachers will need to carefully analyze their existing resources to apply these approaches if this is the case in other grade levels. Teachers, too, may choose to create their writing prompts.

As an overview, table 1 presents a summary of the approaches, purposes they can potentially serve for teachers, and ways to select writing prompts. We share example and nonexample prompts that focus on content within the elementary, middle, and high school grades. Each pair of prompts addresses a common task, and grade-level bands were assigned to the prompts on the basis of the Common Core State Standards for Mathematics (NGA Center and CCSSO 2010) Content Standards. Following this table, we present a discussion of each approach with a scenario observed in everyday practice and formatively assess students' written responses. We encourage readers to consider applications to their teaching and students as they review the various examples across the following sections.

Table 1 Approaches to Identify, Adapt, and Create Writing Prompts to Maximize Mathematical Thinking

Approach and Purposes for Teachers	Writing Prompt Selection	Examples to Apply	Nonexamples to Avoid
Approach 1 Promote students' solution paths. **Purpose** Uncover individual students' preferred solution paths.	• Identify, adapt, and create prompts that promote a variety of approaches. • Beware of prompts that explicitly or implicitly guide and limit students to solve using a particular strategy.	Adding Fractions (Grade 5)	
		Describe an efficient way to add these two sets of fractions: 1/2 + 3/4 and 4 5/6 + 1 1/3.	Use drawings to add the following two sets of fractions: 1/2 + 3/4 and 4 5/6 + 1 1/3.
		Unit Rates (Grades 7–8)	
		Compare the 2019 median hourly wage for the demographically different groups provided. Explain how you can mathematically represent each group's median amount paid versus hours worked.	Draw a graph that compares hours worked versus the amount paid for each group's 2019 median hourly wage.
		2D and 3D Objects (High School Geometry)	
		Identify the two-dimensional figures you would get if you were to cut the following [e.g., triangular prism, cylinder] three-dimensional shapes.	Build a three-dimensional shape using modeling clay and slice it with dental floss to identify the resulting two-dimensional figures.
Approach 2 Go beyond asking students to simply "explain." **Purpose** Better understand students' conceptual mathematical thinking behind procedures.	• Identify, adapt, and create prompts that ask students to address a strategy, concept, or generalization. • Beware of prompts that ask students only to explain the steps they took to compute.	Algorithmic Subtraction (Grades 3–4)	
		You are playing a game with a friend. The person with the lowest score wins. You had 157 points and made a great move to get rid of 129 points. Describe what is happening when you cross out the 5 and record a 4 when subtracting 129 from 157 using the standard algorithm to find how many points you have left.	Subtract 157 – 129 using the standard algorithm. Explain your steps.
		Rectangular Prism Volume (Grade 8)	
		Teach a friend about the meaning of each component of the formula used to calculate the volume of your box of crackers.	Calculate the volume of your box of crackers. List steps to show how you arrived at your solution.
		Finding x-intercepts (High School Algebra)	
		Compare the graph features you see when graphing equations like $x^2 - x - 12 = 0$, $x^2 - x - 30 = 0$, and $x^2 - x - 72 = 0$.	Solve for x in the equation $x^2 - x - 12 = 0$. Explain your answer.

Table 1 Approaches to Identify, Adapt, and Create Writing Prompts to Maximize Mathematical Thinking (cont.)

Approach and Purposes for Teachers	Writing Prompt Selection	Examples to Apply	Nonexamples to Avoid
Approach 3 Prompt students to share their reasoning. **Purpose** Gain a deeper comprehension of the depth of students' mathematical thinking.	• Identify, adapt, and create prompts that push students to share their reasoning. • Beware of prompts that ask students to show their work or computation.	Comparing Numbers (K–Grade 1)	
		Determine if a total of seven cookies is greater than eight cookies. Explain how you know.	Circle the numerals and groups of objects that are greater than eight.
		Solving Linear Equation Pairs (Grade 8)	
		Share two different ways to explain how the point (–1, 2) is a solution to $y = 2x + 4$ and $y = -3x - 1$. Include why you think your reasoning is accurate for both ways.	Identify the point of intersection of the following pairs of linear equations: $y = 2x + 4$ $y = -3x - 1$
		Modeling Exponential Functions (High School Algebra)	
		Convince a friend that you will have enough money to buy your prom ticket in eight weeks if you deposit $1 into a bank account and double your account balance every week.	Create an equation to calculate the amount of money in your bank account if you deposit $1 and your account balance doubles every week.
Approach 4 Have students consider the validity of a given solution. **Purpose** Determine students' understanding of a particular strategy or concept.	• Identify, adapt, and create prompts that address a strategy or concept you need to further assess. • Beware of prompts that do not focus on a specific strategy or concept.	Measuring Length (Grades 2–3)	
		Determine if Eloise correctly measured the length of her dining room table and explain why [on the basis of an accompanying drawing showing spaces between her iteration of the ruler].	Use your ruler to measure the length of the dining room table shown in this picture.
		Proportionality (Grade 7)	
		Kayden used the following table to compare the ingredient amounts for two different recipes and concluded they are proportional. Do you agree or disagree? Why?	Determine if the ingredients in the two recipes are proportionate.
		Circle Equations (High School Geometry)	
		Jae drew the circle whose equation is $(x - 3)^2 + (y - 2)^2 = 9$ and said that the circle area represents a food desert area on the map. Jae said the center is (3, 2) and the radius is 9. Do you agree or disagree with Jae's solution? Explain why or why not.	Jae drew the circle whose equation is $(x - 3)^2 + (y - 2)^2 = 9$ and said that the circle area represents a food desert area on the map. Jae said the center is (3, 2) and the radius is 9. Highlight the mistake Jae made in the statement above.

Table 1 Approaches to Identify, Adapt, and Create Writing Prompts to Maximize Mathematical Thinking (cont.)

Approach and Purposes for Teachers	Writing Prompt Selection	Examples to Apply	Nonexamples to Avoid
Approach 5 Have students debate the validity of two given solutions. **Purpose** Assess students' reasoning of correct solutions and common misconceptions.	• Identify, adapt, and create prompts that address correct solutions and common misconceptions. • Beware of prompts that do not allow you to comprehensively assess students' understanding.	Angle Measurement (Grade 4)	
		[Provide congruent angles, with the green one having longer arms.] Aneesha thinks the blue and green angles they drew in their art project are congruent. Gavin states the green one is greater. Convince them who is right by sharing how you know.	Measure the following angles. [Provide various angles for students to measure.]
		Absolute Value (Grade 6)	
		Alicia and Aloys are debating the solution to $\lvert 30 \rvert$. Alicia thinks the answer is −30, and Aloys says it is 30. Write an email to the pair to convince them who is correct, and why.	Find the absolute value of 30.
		Expected Value (High School Statistics)	
		Jaylin thinks that the expected value of rolling a die is 3.5. Alvyn thinks that is impossible and says the expected value is 3. Whom do you agree with, and why?	Calculate the expected value when rolling a die.

Approach 1: Promote Students' Solution Paths

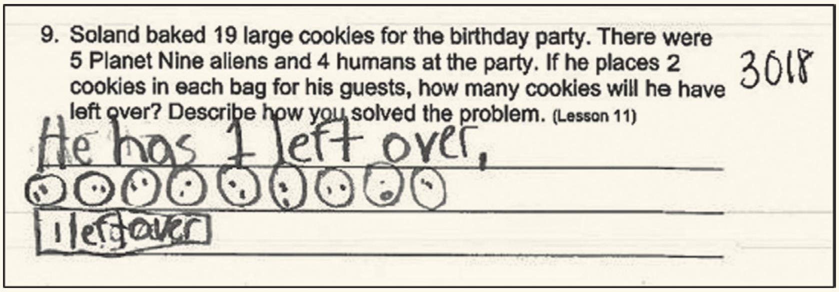
Envision any given number of the student activity sheets you use that provide one or more suggestions for how to solve the problems being posed. They may include a thinking bubble describing how to solve the problem, an example that has been worked out, or prompts to use a particular model (e.g., "Draw a picture to . . ." and "Use a table to . . ."). Although some of these approaches might be considered "suggestions," students can interpret them as directives even if a teacher encourages them to use any strategy they wish. Students are keen to pick up on explicit or implicit suggestions, such as the need to use a number line or coordinate plane with all problems listed on pages with these images. Instead, strip away these directives and hints to provide students with the opportunity to determine their personal solution paths. The placement and inclusion of these tips inherently have students follow someone else's lead, such as curriculum authors who already have engaged in mathematical thinking because they reasoned that to be a viable solution path.

In allowing students to share their approaches, teachers will be positioned to formatively assess students' mathematical thinking more comprehensively and individually. If a particular strategy already has been attended to in class, then teachers would get a sense of the extent to which students have applied it. As figure 1 shows, Dominic demonstrated that he understood the concept of division that was discussed by the class, yet the extent to which he did is unclear in his writing because his description refers to a visual representation.

If the strategy has not yet been addressed during class, then students' solutions can be analyzed concerning their complexity, which can indicate how well they understand the concepts and apply their thinking in a variety of ways. Jayney (see figure 2) relied on an equation, graph, and table as part of her written argument, suggesting she comprehensively understood multiple algebraic representations and the connections among them.

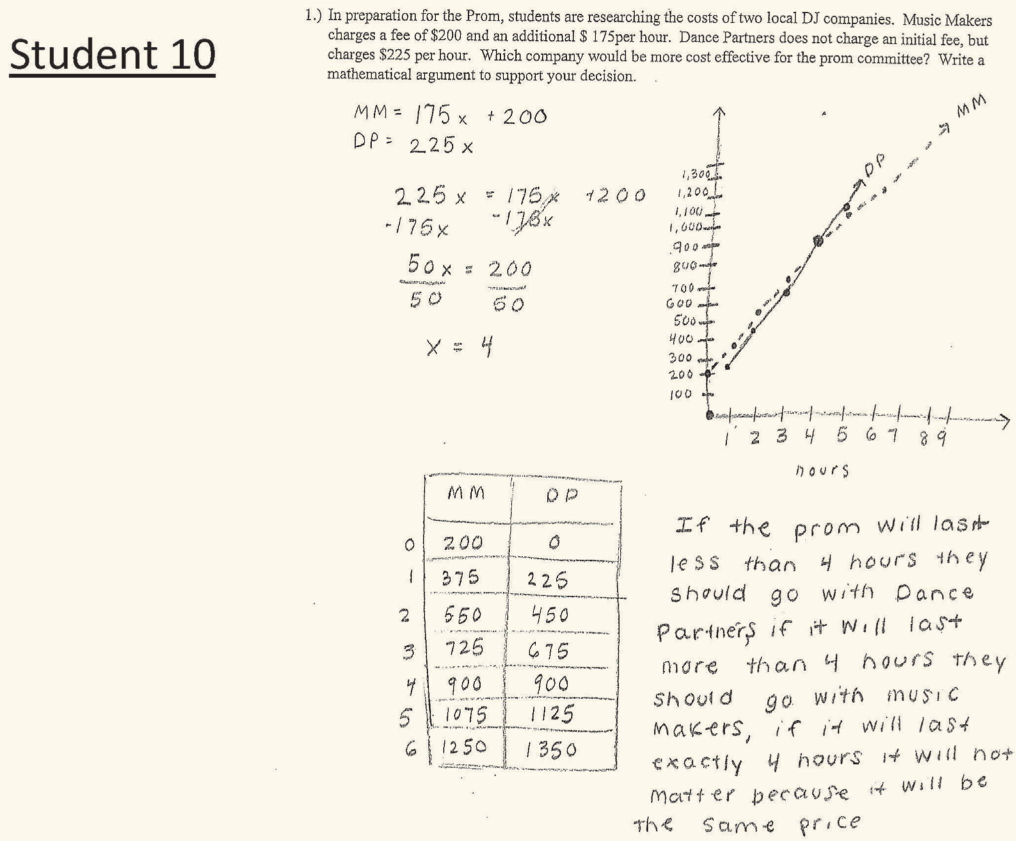
Altogether, prompting students to share their preferred solution paths in writing can provide teachers with insights into the extent to which the class and individuals understand key ideas that, in turn, can inform their future instructional plans. For example, an elementary student who realizes that repeatedly adding is a more efficient way to compute the total compared to counting individual objects is on their way to understanding a fundamental multiplication concept. A middle or high schooler who needs to solve for *y*

Fig. 1



Dominic used a model to solve the division problem and provided a sentence to share his answer. His teacher planned to follow up with him to share more details about his solution path. (Student work completed through Thinking Like Mathematicians: Challenging All Grade 3 Students.)

Fig. 2



Jayney relied on an equation, graph, and table without being prompted to do so to argue that the prom committee can hire either DJ company. (Retrieved from University of Connecticut Bridging Practices among Connecticut Mathematics Educators, *n.d.)*

when x is zero to determine the y-intercept has grasped the meaning of this concept but has not yet made a connection to how it is represented in a linear equation.

Approach 2: Go beyond Asking Students to Simply "Explain"

Looking closely at your existing writing prompts on student pages, you may notice that students frequently are asked to simply "explain" and are sometimes guided to also include a drawing or other visual representation, an equation, or words. Students might interpret such writing prompts to mean that they should record only their computations or a superficial level of detail. The challenge for teachers trying to formatively assess their class is that they need to infer students' thinking on the basis of the steps or basic information students wrote down.

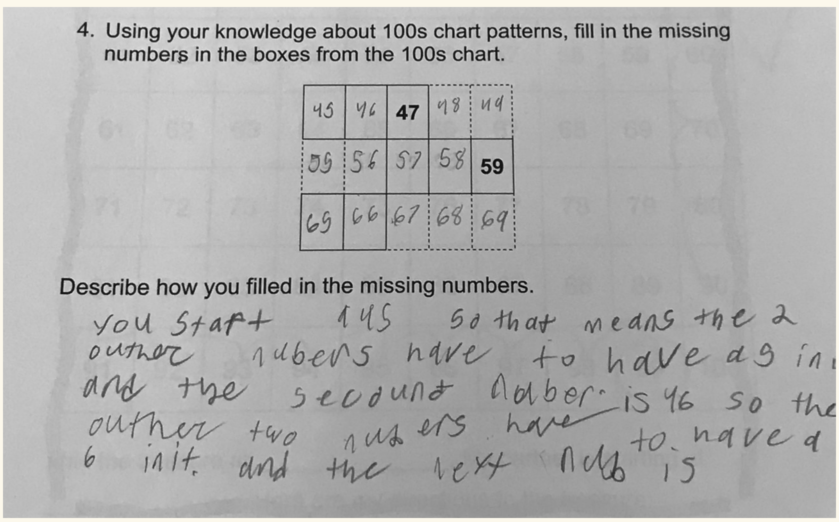
Writing prompts that have students explain their understanding of, for instance, a definition, a concept, or an observation allow teachers to evaluate the depth and comprehensiveness of each student's knowledge of these ideas. Such prompts can focus on a strategy (e.g., "Tell a friend how you solved . . ."), concept (e.g., "Describe the meaning of . . ."), and generalization (e.g., "Explain the patterns you noticed in . . .") (Casa et al. 2019). Notice how the follow-up prompt to question 4 (see figure 3) extends beyond solely asking students to complete the missing values in the hundred chart. Dahlia's response in the chart and her writing conveyed that she understood the pattern of ascending numbers in a hundred chart and that numbers listed in the same column have the same digit in the ones place.

Overall, asking students to go beyond just listing computational steps to explain their understanding and reasoning provides a window into the extent to which they realize the underlying mathematical concepts. An elementary student who notes that a square "lays flat" may not yet realize that describing it as a "two-dimensional shape" or a "polygon" is a more precise way to do so. Alternatively, this student may incorrectly think that the shape must be positioned so that a side is parallel to the bottom of the paper and, thus, is not a square if it has been rotated. A middle schooler who states that they found an equivalent fraction by multiplying by a representation of 1 rather than writing that they "multiplied by a fraction with the same top and bottom number" has demonstrated a more advanced understanding of the concept behind the identity property of multiplication.

Approach 3: Prompt Students to Share Their Reasoning

Imagine prompts that ask students to "show their work." Students often first rely on exploratory writing to solve the problem (Casa et al. 2016) before they consider how to present their ideas to a reader, including their teacher. Because the purpose of exploratory writing positions students as their own audience, which often can mean distinct ideas are recorded randomly on their papers,

Fig. 3

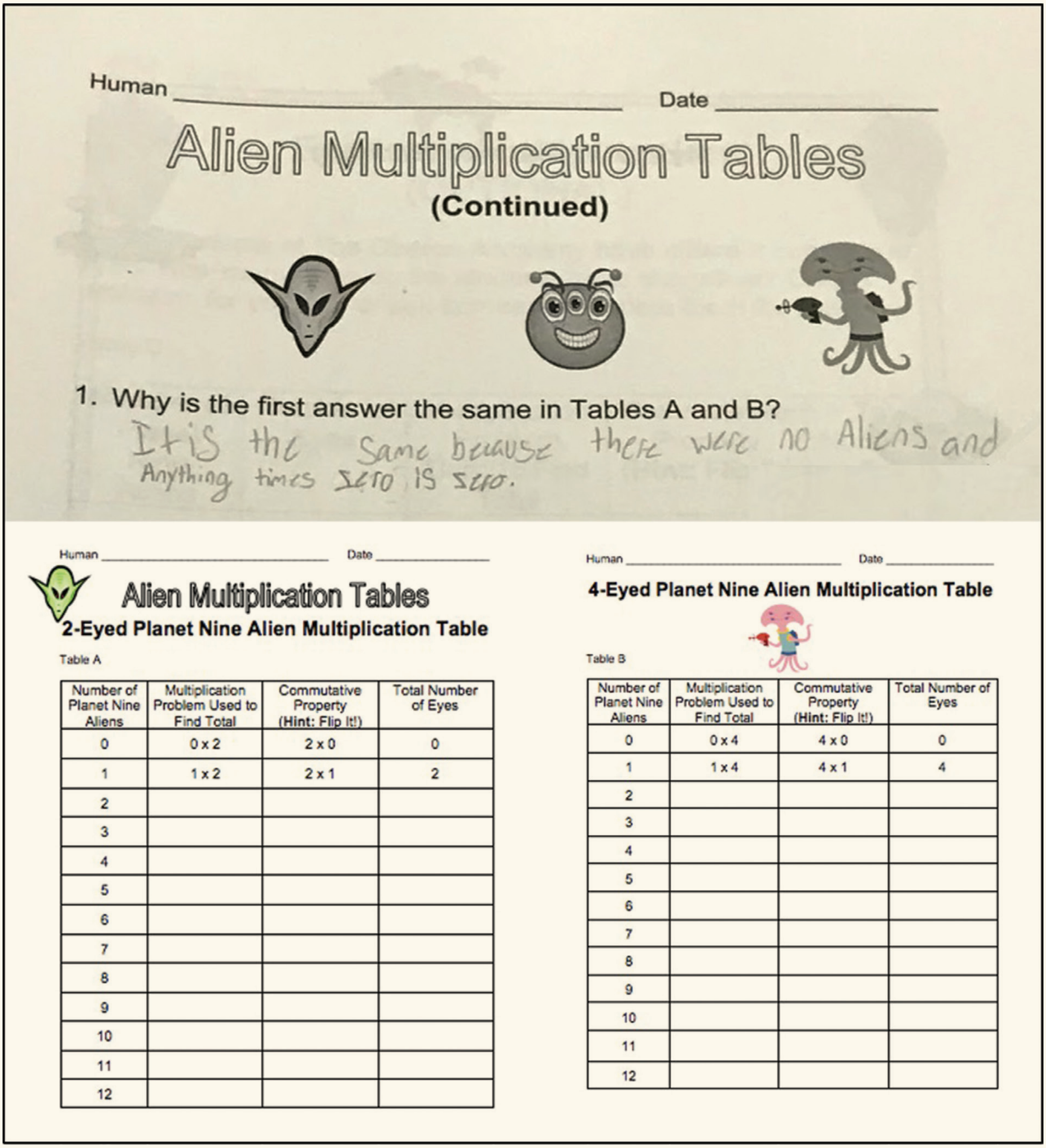


Dahlia shared her reasoning reflecting the patterns used to complete the hundred chart. (Student work completed through Thinking Like Mathematicians: Challenging All Grade 3 Students).

following each student's logical flow can be challenging for teachers when they later read a stack of class papers.

Writing prompts that explicitly ask students to share their reasoning based on the argument students relied on to solve a problem have the potential to provide a teacher with insights into how students approached the solution and how to assess their sophistication. For example, as figure 4 shows, students are asked to describe why the first answer in tables A and B are the same after they had a class discussion about the role of zero. Although

Fig. 4



Human ______________ Date ______________

Alien Multiplication Tables

(Continued)

1. Why is the first answer the same in Tables A and B?

It is the Same because there were no Aliens and Anything times zero is zero.

Human ______________ Date ______________

Alien Multiplication Tables

2-Eyed Planet Nine Alien Multiplication Table

Table A

Number of Planet Nine Aliens	Multiplication Problem Used to Find Total	Commutative Property (Hint: Flip It!)	Total Number of Eyes
0	0 x 2	2 x 0	0
1	1 x 2	2 x 1	2
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			

Human ______________ Date ______________

4-Eyed Planet Nine Alien Multiplication Table

Table B

Number of Planet Nine Aliens	Multiplication Problem Used to Find Total	Commutative Property (Hint: Flip It!)	Total Number of Eyes
0	0 x 4	4 x 0	0
1	1 x 4	4 x 1	4
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			

Shane shared part of his reasoning as to why there are zero eyes in both tables A and B when there are zero aliens. (Student work completed through Thinking Like Mathematicians: Challenging All Grade 3 Students.)

Shane indicated that when zero is one of the factors being multiplied, it means that the product will always be zero and that there were zero aliens, his writing is unclear to what extent he understands the underlying concept that other students connected to the theme that highlighted that it is not possible to repeatedly add zero.

In general, students can be asked to write an argument using prompts like this: “How do you know?”; “Convince ___ that . . .”; and “Do you agree or disagree? Tell why” (Firmender, Casa, and Colonnese 2017). Doing so regularly can give teachers insights into the comprehensiveness of their students’ reasoning. For example, an elementary student who reasons that one-third is greater than one-half and includes a drawing with unequal-sized pieces likely does not yet realize this key mathematical concept when comparing fractions and the symbolic representation and meaning of the denominator. A high school student who writes an argument defending the fact that despite rolling two dice 20 times and surprisingly rolling double 1’s six times has not yet realized the theoretical probability of having the dice sum to 2 is much less than having them summing to 7.

Approach 4: Have Students Consider the Validity of a Given Solution

Think about a time when you needed to emphasize a particular strategy, but none or very few of your students addressed it in their writing. Without having evidence from all students, teachers cannot assess their class’s collective understanding even if it was a perspective that was discussed. Introducing a response to a prompt that shares a current or former students’ solution strategy has students attend to it without usurping their currently preferred path. This approach also allows students to question others’ mathematical arguments and encourages them to establish correct solution pathways (Kersaint 2015). A written response like Isabela’s (see figure 5) additionally represents an opportunity for the class to consider the connection between concepts and procedures from a student perspective. Such insights contribute to teachers’ determination of the extent of each of their students’ understanding.

Altogether, having students consider the validity of a given solution fosters a learning environment where students realize the ongoing expectation of communicating one’s mathematical reasoning in writing (Firmender, Casa, and Colonnese 2017). Sharing work anonymously further provides the opportunity for teachers to present a doctored sample that targets the content and solution strategies they deem important for their class to consider. For example, elementary students who insist on relying on a traditional multidigit subtraction algorithm would benefit from considering the use of an open number line to find the difference between 76 and 100. Likewise, students in high school would notice how someone else relied on certain algebraic manipulations and strategies to solve systems of equations.

Approach 5: Have Students Debate the Validity of Two Given Solutions

Reflect on a time within an instructional unit when you needed to ensure that students fully comprehended a key concept. Although students previously may have shared a variety of approaches to solving problems during class discussions and through writing, teachers engaged in the formative assessment process might question the extent to which individual students fully grasped core understandings. Having students debate in writing about the viability of two proposed solutions to a problem supplies teachers with these missing insights.

Presenting a combination of one correct solution, two correct solutions, and two misconceptions, as suggested by Bostiga and colleagues (2016), will help ensure that students are responsible for their mathematical thinking. Otherwise, students might instead automatically assume that the solution must be a faulty one if they realized a pattern across similar types of prompts. Then, teachers can conclude that students authentically are cognitively engaged and share their understanding as they formatively assess their class. Students may choose to justify their thinking about a correct solution, such as when Lashon specified which of the solutions he agreed with and used models to justify his thinking (see figure 6).

Students may choose to disprove an incorrect solution in addition to presenting their reasoning as to why another one is correct. Figure 7 depicts a sample a teacher composed that combines several students’ responses into one that the class reviewed.

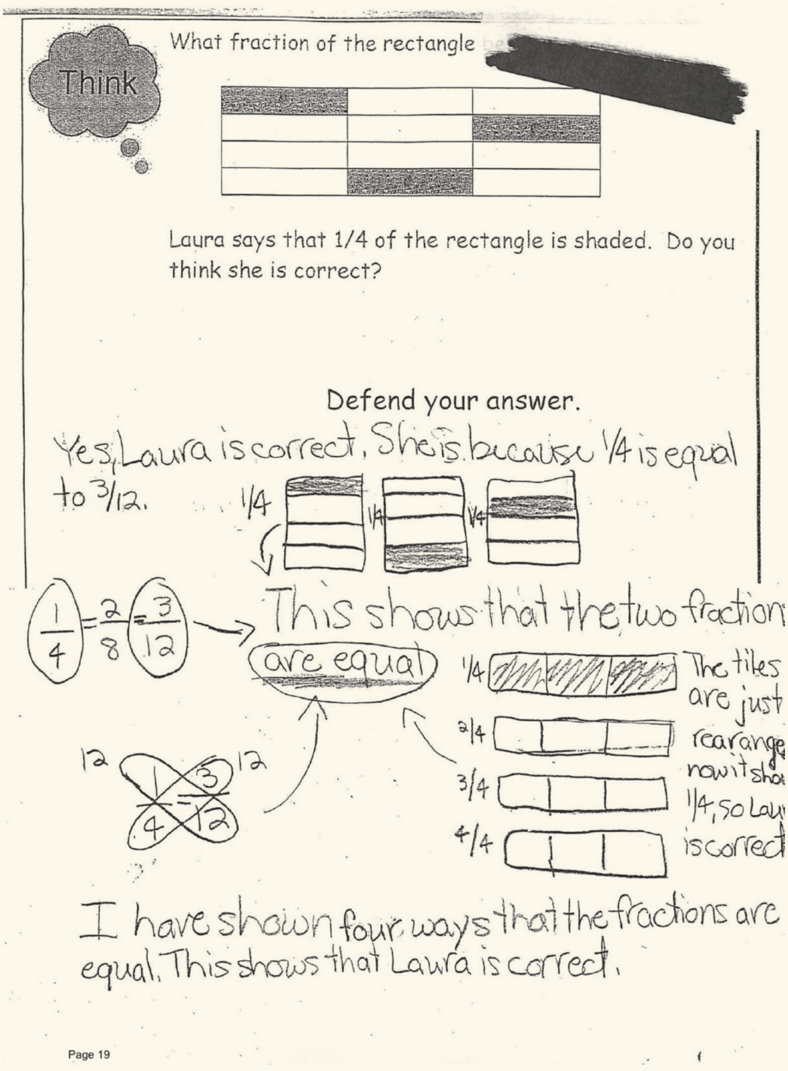
In general, debatable prompts that present common misconceptions and errors arising in their class in combination with correct ideas can supply teachers with invaluable insights into their students’ depth of knowledge over time. These prompts also induce students to build a greater understanding of the mathematical content (Kersaint 2015). For example, elementary students first learning about decimals need to understand the role of the zeros written between the decimal point and digits to its right, such as the zeros in the tenths and hundredths place in 0.008, which are not recorded with integers (e.g., we conventionally record “8” not “008”). Similarly, secondary students working with signed numbers particularly can be challenged by computations with negative numbers.

CONCLUSION

We have collaborated with numerous teachers who have relied on writing prompts using the five aforementioned approaches, and they have shared how much more valuable they were to understand the depth of their students’ mathematical thinking. These teachers believed they could ground their instructional decisions in ways that met their students where they were in their current understanding. Although some students initially did not write down as much as their teachers desired when they implemented these approaches, not asking students to write in ways that represent their mathematical thinking

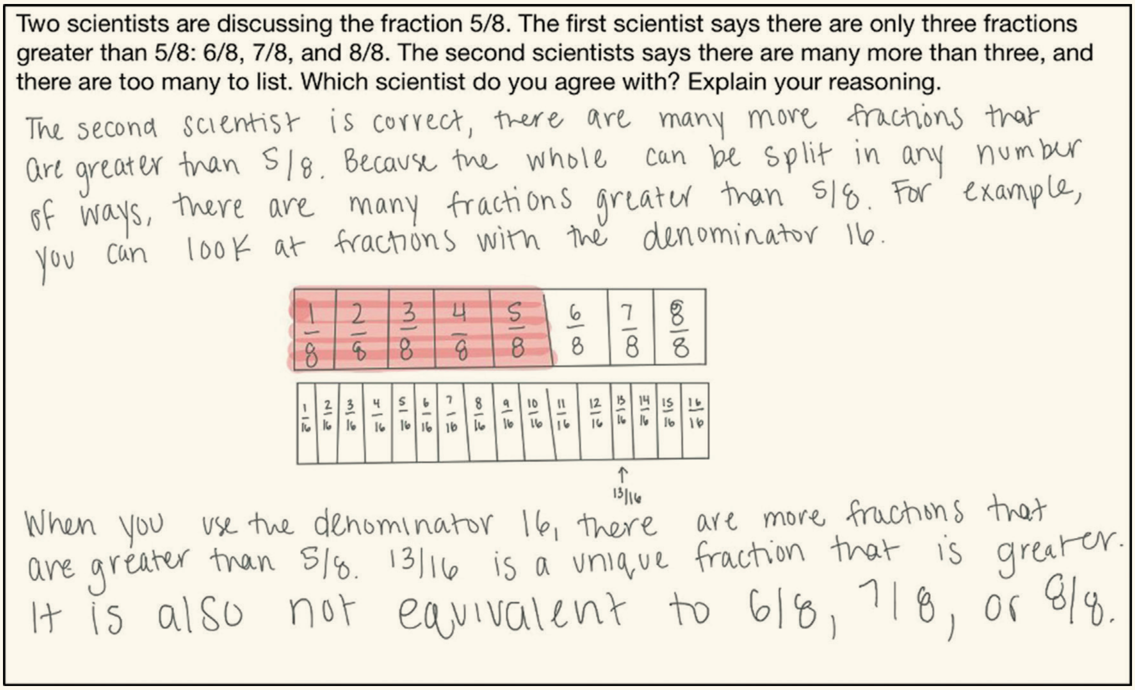
Fig. 5

Student 1



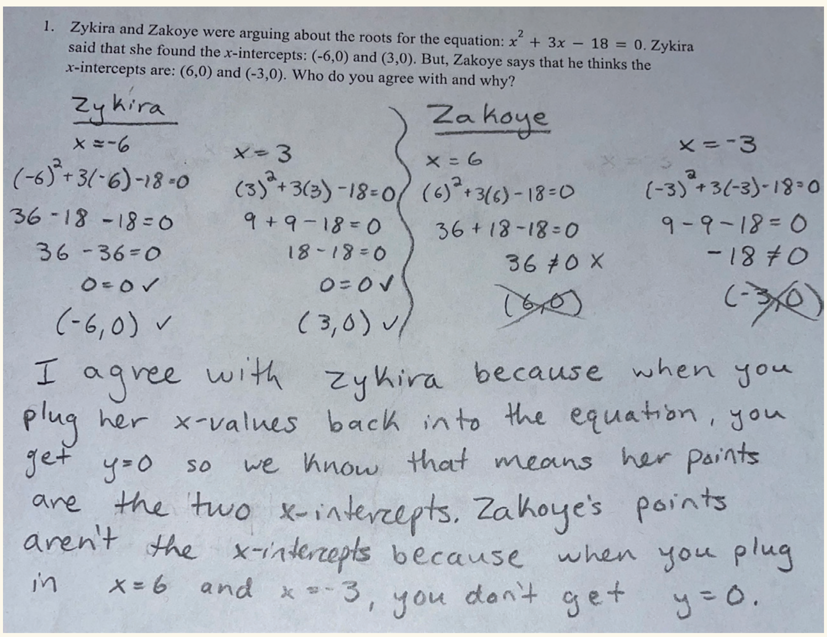
Isabella backed her claim using drawings to convey the concept of a fraction and connect it to some procedures. (Retrieved from University of Connecticut Bridging Practices among Connecticut Mathematics Educators, *n.d.)*

Fig. 6



Lashon used an example to disprove the first scientist's statement and display his comprehensive understanding of fractions. (Response used with teachers field testing a unit in the grant Thinking Like Mathematicians: Challenging All Grade 3 Students.)

Fig. 7



This response is a comprehensive one because it attends to the correct and incorrect solutions. Samples like this one can help the class realize how mathematical writers may choose to present their argument, which would allow teachers to formatively assess an individual student's understanding in a more comprehensive manner.

meant teachers were limited in their efforts beforehand. We believe that making the effort to identify, adapt, or create writing prompts that teachers assign students each day has great promise. Doing so takes no extra instructional time, allows teachers to home in on their class's and individual students' needs that will inform their future instructional decision making, and has the potential to streamline instruction across time. Ultimately, providing students with the opportunity to share more about their understanding honoring their mathematical approaches presents them with more equitable opportunities to illuminate their thinking and knowledge (Kersaint 2015; Ladson-Billings 1997).

REFERENCES

Bostiga, Shannon E., Michelle L. Cantin, Cristina V. Fontana, and Tutita M. Casa. 2016. "Moving Math in the Write Direction." *Teaching Children Mathematics* 22, no. 9 (May): 546–54.

Casa, Tutita M., Jeffrey M. Choppin, and Judit N. Moschkovich. 2020. "Crafting a Research Agenda for Writing to Reason Mathematically." *Research Agenda for Mathematical Writing*. s.uconn.edu/WritingInMath.

Casa, Tutita M., Janine M. Firmender, June Cahill, Fabiana Cardetti, Jeffrey M. Choppin, Jeremy Cohen, . . . Ryley Zawodniak. 2016. "Types of and Purposes for Elementary Mathematical Writing: Task Force Recommendations." https://mathwriting.education.uconn.edu/.

Casa, Tutita M., Juliana R. MacSwan, Kara E. LaMonica, Madelynn W. Colonnese, and Janine M. Firmender. 2019. "An Analysis of the Amount and Characteristics of Writing Prompts in Grade 3 Mathematics Student Books." *School Science and Mathematics* 119, no. 4 (April): 176–89.

Cole, Shelby, Lisa Rubestein, Tutita M. Casa, Cindy Gilson, Micah N. Bruce-Davis, E. Jean Gubbins, Alexis Melendez, Hannah F. Brown, Siamak Vahidi, Rachel McAnallen, and Stacy Hayden. 2019. *If Aliens Taught Algebra: Multiplication and Division Would Be out of This World!* 3rd ed. Storrs, CT: Renzulli Center for Creativity, Gifted Education, and Talent Development.

Firmender, Janine M., Tutita M. Casa, and Madelynn W. Colonnese. 2017. "Write On: Reasoning through Mathematical Writing." *Teaching Children Mathematics* 24, no. 2 (October): 84–92.

Gutiérrez, Rochelle. 2018. "The Need to Rehumanize Mathematics." In *Rehumanizing Mathematics for Black, Indigenous, and Latinx Students*, edited by Imani Goffney, Rochelle Gutiérrez, and Melissa Boston, pp. 1–10. Reston, VA: National Council of Teachers of Mathematics.

Kersaint, Gladis. 2015. *Orchestrating Mathematical Discourse to Enhance Student Learning: Creating Deeper Learning by Engaging Students in Math Discussion*. North Billerica, MA: Ready Curriculum Associates. https://ttaconline.org/Document/zxbIhX_YCJNP0qvIYsAjT0x-qdzE3VlX/WP-Curriculum_Associates%E2%80%93Orchestrating_Mathematical_Discourse.pdf0.pdf.

Ladson-Billings, Gloria. 1997. "It Doesn't Add Up: African American Students' Mathematics Achievement." *Journal for Research in Mathematics Education* 28, no. 6 (December): 697–708.

National Council of Teachers of Mathematics (NCTM). 2000. *Principles and Standards for School Mathematics*. Reston, VA: NCTM.

National Council of Teachers of Mathematics (NCTM). 2014. *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: NCTM.

National Council of Teachers of Mathematics (NCTM), National Council of Supervisors of Mathematics (NCSM), and Association of State Supervisors of Mathematics (ASSM). 2021. *Continuing the Journey: Mathematics Learning 2021 and Beyond*. Research brief. NCTM, NCSM, and ASSM. https://www.nctm.org/mathematics2021/.

National Governors Association Center for Best Practices (NGA Center) and Council of Chief State School Officers (CCSSO). 2010. *Common Core State Standards for Mathematics*. Washington, DC: NGA Center and CCSSO. http://www.corestandards.org.

The Regents of the University of California. 2021. "Understanding the Formative Assessment Process." Tools for Teachers Smarter Balanced. https://portal.smarterbalanced.org/library/en/formative-assessment-process.pdf.

Renzulli, Joseph S., and Nicole Waicunas. 2016. "An Infusion-Based Approach to Enriching the Standards-Driven Curriculum." In *Reflections on Gifted Education*, edited by Joseph S. Renzulli and Nicole Waicunas, pp. 411–28. Waco, TX: Prufrock Press.

University of Connecticut. n.d. "Argumentation Resource Packets." *Bridging Practices among Connecticut Mathematics Educators.* Accessed March 3, 2022. https://bridges.education.uconn.edu/argumentation-resource-packets-2/.

ACKNOWLEDGMENTS

This work was supported by the Jacob K. Javits Gifted and Talented Students Education Program, United States Department of Education PR/Award No. S206A170023, Thinking Like Mathematicians: Challenging All Grade 3 Students.

